

Solution / marking scheme – Neutron Stars (10 points)

General rules

- In the following, “coefficients” refer to the numerical factors and do not include parameters.

Part A. Mass and stability of nuclei (2.5 points)

A.1 (total 0.9 pt)

(0.9 pt)

$$A = 50$$

- No reduction if $A = 5.0 \times 10^1$.
- 0.8 pt if the value is in the range 49.5–50.4.

— partial points —

$$(0.2 \text{ pt}) \quad \frac{B}{A} = a_V - a_S A^{-1/3} - \frac{a_C}{4} A^{2/3} \quad (\text{A.1.1})$$

- No reduction if the difference from (A.1.1) is only the overall coefficient. This rule is applied throughout.

$$(0.1 \text{ pt}) \quad \frac{d(B/A)}{dA} = 0 \quad (\text{A.1.2})$$

$$(0.2 \text{ pt}) \quad \frac{a_S}{3} A^{-4/3} - \frac{a_C}{6} A^{-1/3} = 0 \quad (\text{A.1.3})$$

- Points for (A.1.2) are given if (A.1.3) is stated although (A.1.2) is not explicitly written.

$$(0.2 \text{ pt}) \quad A = \frac{2a_S}{a_C} \quad (\text{A.1.4})$$

- 0.7 pt is given if the correct expression for A appears even if the intermediate steps are not fully written.

A.2 (total 0.9 pt)

(0.9 pt)

$$Z^* = 79$$

- No reduction if $Z^* = 78$.
- 0.8 pt if the value is in the range 77.5–79.4.

— partial points —

$$(0.3 \text{ pt}) \quad -2a_C \frac{Z^*}{A^{1/3}} - 4a_{\text{sym}} \frac{2Z^* - A}{A} = 0 \quad (\text{A.2.1})$$

$$(0.4 \text{ pt}) \quad Z^* = \frac{1}{1 + \frac{a_C}{4a_{\text{sym}}} A^{2/3}} \cdot \frac{A}{2} \quad (\text{A.2.2})$$

- No reduction if $a_C/4a_{\text{sym}}$ is replaced by the numerical value in the range 0.007–0.008.

A.3 (total 0.7 pt)

(0.7 pt)

$$C_{\text{fission}} = 0.70$$

- No reduction if $C_{\text{fission}} = 0.7$.

— partial points —

$$(0.3 \text{ pt}) \quad a_S \left[A^{2/3} - 2 \left(\frac{A}{2} \right)^{2/3} \right] + a_C \left[\frac{Z^2}{A^{1/3}} - 2 \frac{(Z/2)^2}{(A/2)^{1/3}} \right] > 0 \quad (\text{A.3.1})$$

- No point if a_V is not canceled.

$$(0.2 \text{ pt}) \quad \frac{Z^2}{A} > \frac{2^{1/3} - 1}{1 - 2^{-2/3}} \cdot \frac{a_S}{a_C} \quad (\text{A.3.2})$$

- Points for (A.3.1) are given if (A.3.2) is stated although (A.3.1) is not explicitly written.
- The coefficient may have different expressions, e.g., with $x = 2^{1/3}$,

$$\frac{x-1}{1-x^{-2}} = \frac{x^2}{1+x} = \frac{x}{1+x^{-1}} = \dots = 0.702414\dots$$

Part B. Neutron star as a gigantic nucleus (1.5 points)

B.1 (total 1.5 pt)

(0.8 pt)

$$a_{\text{grav}} = 6 \times 10^{-37} \text{ MeV}$$

- No reduction if the unit is not written.
- 0.7 pt if only the order of magnitude is correct.

— partial points —

$$(0.4 \text{ pt}) \quad a_{\text{grav}} = \frac{3}{5} \frac{Gm_N^2}{R_0} \quad (\text{B.1.1})$$

$$(0.2 \text{ pt}) \quad a_{\text{grav}} = \frac{3}{5} \frac{\hbar c m_N^2}{R_0 M_P^2} \quad (\text{B.1.2})$$

- Points for (B.1.1) are given if (B.1.2) is stated although (B.1.1) is not explicitly written.
- No reduction if \hbar is mistyped.

(0.7 pt)

$$A_c = 4 \times 10^{55}$$

- No reduction for $A_c = 5 \times 10^{55}$.
- 0.6 pt if only the order of magnitude is correct.

— partial points —

$$(0.2 \text{ pt}) \quad a_V A - a_{\text{sym}} A + a_{\text{grav}} A^{5/3} > 0 \quad (\text{B.1.3})$$

$$(0.3 \text{ pt}) \quad A_c = \left(\frac{a_{\text{sym}} - a_V}{a_{\text{grav}}} \right)^{3/2} \quad (\text{B.1.4})$$

- Points for (B.1.3) are given if (B.1.4) is stated although (B.1.3) is not explicitly written.

Part C. Neutron star in a binary system (6.0 points)

C.1 (total 1.0 pt)

(1.0 pt)

$$\Delta\tau_{\text{II}} = \left(1 - \frac{\Delta\phi}{c^2}\right) \Delta\tau_{\text{I}}$$

- No points if the coefficient is wrong.

— partial points —

$$(0.3 \text{ pt}) \quad v^2 = 2g\Delta h = 2\Delta\phi \quad \text{or} \quad v = \sqrt{2\Delta\phi} \quad (\text{C.1.1})$$

$$(0.5 \text{ pt}) \quad \Delta\tau_{\text{II}} = \sqrt{1 - v^2/c^2} \Delta\tau_{\text{I}} \quad \text{or} \quad \Delta\tau_{\text{II}} = \sqrt{1 - 2\frac{\Delta\phi}{c^2}} \Delta\tau_{\text{I}} \quad (\text{C.1.2})$$

- Points for (C.1.1) are given if (C.1.2) is stated although (C.1.1) is not explicitly written.

C.2 (total 1.8 pt)

(1.8 pt)

$$\Delta t = \frac{2GM_{\text{WD}}}{c^3} \log\left(\frac{4|x_N|x_E}{d^2}\right)$$

- No reduction if 4 is missing in log.
- No reduction if $|x_N|$ is written as $-x_N$.
- 0.1 pt is subtracted if the modulus in $|x_N|$ is missing.
- No points if other coefficients are wrong.

— partial points —

$$(0.5 \text{ pt}) \quad t_{\text{E-N}} = \int_{x_N}^{x_E} \frac{dx}{c_{\text{eff}}(x)} \quad \text{or} \quad \Delta t_{\text{E-N}} = \frac{\Delta x}{c_{\text{eff}}(x)} \quad (\text{C.2.1})$$

$$(0.4 \text{ pt}) \quad t_{\text{E-N}} \simeq \frac{1}{c} \int_{x_N}^{x_E} dx \left(1 + \frac{2GM_{\text{WD}}}{c^2\sqrt{x^2 + d^2}}\right) \quad (\text{C.2.2})$$

- 0.1 pt is subtracted if the coefficient is wrong.

$$(0.3 \text{ pt}) \quad \Delta t = \frac{2GM_{\text{WD}}}{c^3} \int_{x_N}^{x_E} \frac{dx}{\sqrt{x^2 + d^2}} \quad (\text{C.2.3})$$

$$(0.3 \text{ pt}) \quad \text{Inside the logarithm: } \sqrt{x_N^2 + d^2} + x_N \simeq \frac{d^2}{2|x_N|} \quad \text{and} \quad \sqrt{x_E^2 + d^2} - x_E \simeq \frac{d^2}{2x_E} \quad (\text{C.2.4})$$

C.3 (total 1.8 pt)

(1.8 pt)

$$\Delta t_{\max} - \Delta t_{\min} = \frac{2GM_{\text{WD}}}{c^3} \log(4/\varepsilon^2)$$

- No reduction if log is written as ln.

———— partial points ————

$$(0.6 \text{ pt}) \quad \Delta t_{\max} = \frac{2GM_{\text{WD}}}{c^3} \log(4x_E/L\varepsilon^2) \quad (\text{C.3.1})$$

- No subtraction points if the factor in log is different but consistent with that in C.2.
- 0.1 pt is subtracted if the coefficient is wrong.

$$(0.2 \text{ pt}) \quad \text{Because of } x_N > 0 \text{ the approx. in log is changed: } x_N + \sqrt{x_N^2 + d^2} \simeq 2L \quad (\text{C.3.2})$$

$$(0.4 \text{ pt}) \quad \Delta t_{\min} = \frac{2GM_{\text{WD}}}{c^3} \ln(x_E/L) \quad (\text{C.3.3})$$

- Points for (C.3.2) are given if (C.3.3) is stated although (C.3.2) is not explicitly written.
- 0.1 pt is subtracted if the coefficient is wrong.

$$(0.3 \text{ pt}) \quad \text{Points are given if } L \text{ and } x_E \text{ dependence is canceled in log.} \quad (\text{C.3.4})$$

C.4 (total 0.8 pt)

(0.8 pt)

$$M_{\text{WD}}/M_{\odot} = 0.5$$

- No reduction if the value is in the range 0.4–0.5.

———— partial points ————

$$(0.2 \text{ pt}) \quad \varepsilon^2 \simeq 2 \times (1 - 0.99989) = 0.00022 \quad (\text{C.4.1})$$

$$(0.2 \text{ pt}) \quad \text{From the given graph, } \Delta t_{\max} - \Delta t_{\min} \approx 50 \mu\text{s} \quad (\text{C.4.2})$$

- No reduction if the value from the graph is in the range 40–50 μs .

$$(0.2 \text{ pt}) \quad M_{\text{WD}}/M_{\odot} \simeq 5/\ln(4/\varepsilon^2) \quad (\text{C.4.3})$$

- No reduction if the numerator is in the range 4–5.

C.5 (total 0.4 pt)

(0.4 pt)

$$p = -\frac{3}{2} \quad \text{or} \quad -1.5$$

- No points if the sign is wrong.

—— partial points ——

(0.3 pt) $R^3\omega^2 = (\text{const.})$

(C.5.1)

C.6 (total 0.2 pt)

(0.2 pt)

The most appropriate profile is (b).