Mass Measurement

Revision in Part A.3 such that
$-0.8 \leqq a \leqq -0.3$ mm (0.1)
instead of $-0.08 \leqq a \leqq -0.03$ mm (0.1).

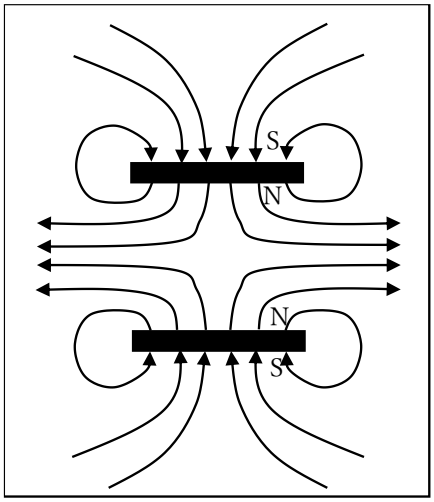
Write down the numbers 0 to 9 in

0	1	2	3	4	5	6	7	8	9
0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*

No points

Part A: Hooke's law and electromagnetic forces (2.4 points)

A.1 (0.4pt)



Magnetic field lines have arrows from N to S (0.1)
At least one end comes from a magnet (0.1)
Multiple horizontal lines near the edge of the magnet gap (0.1)
No contradictions such as asymmetry, crossing or branching (0.1)

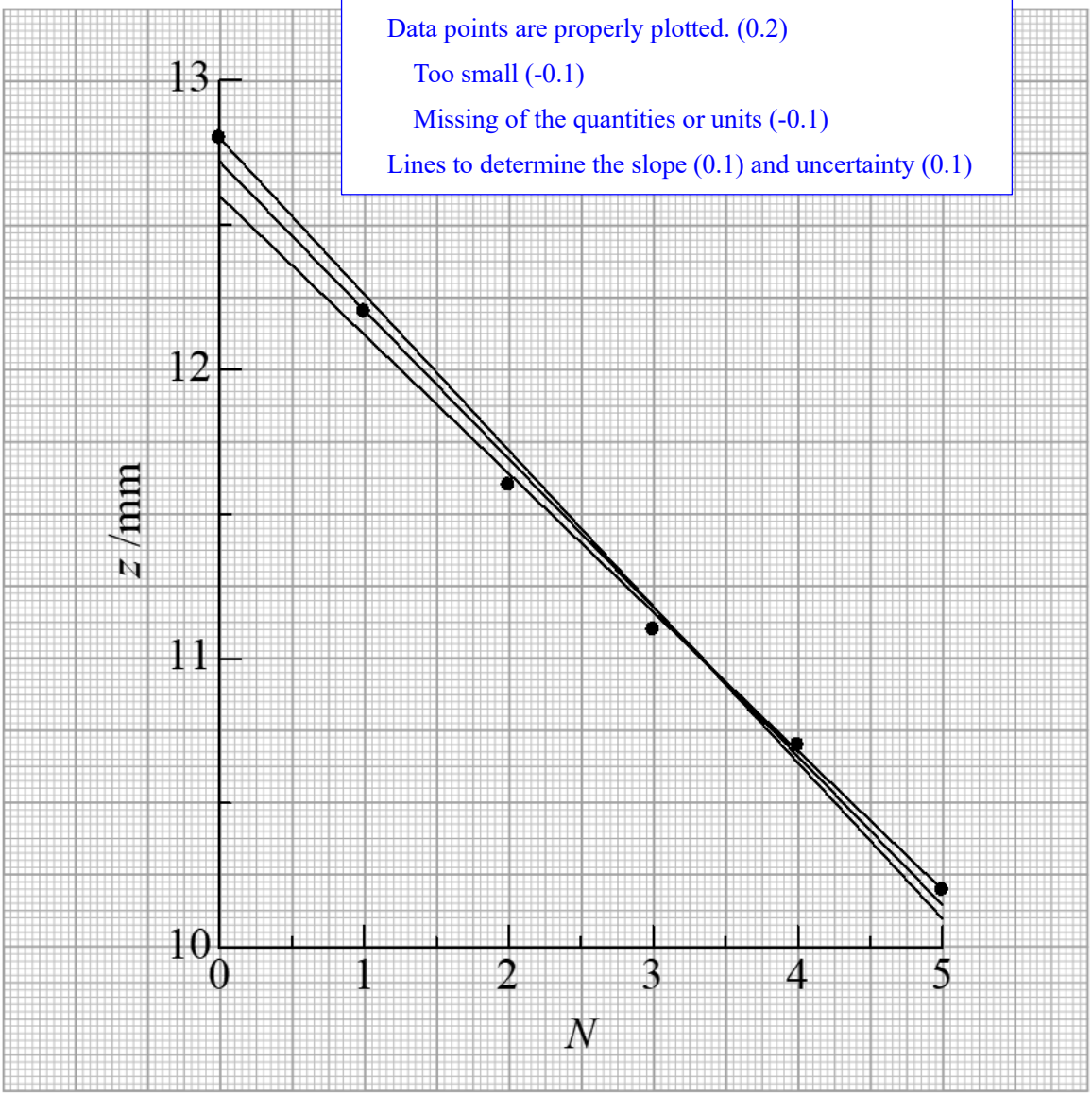
A.2 (0.6pt)

Missing measurement points (-0.1 each)

N	z /mm	I /A
0	12.8	0
1	12.2	0.103
2	11.6	0.213
3	11.1	0.323
4	10.7	0.423
5	10.2	0.524

A.3 (0.7pt)

Graph (max 0.4)
- Data points are properly plotted. (0.2)
  - Too small (-0.1)
  - Missing of the quantities or units (-0.1)
- Lines to determine the slope (0.1) and uncertainty (0.1)



$$a = \frac{\Delta z}{\Delta N} = \frac{10.15-12.70}{5} = -0.51$$

$$a_+ = \frac{10.20-12.60}{5} = -0.48$$

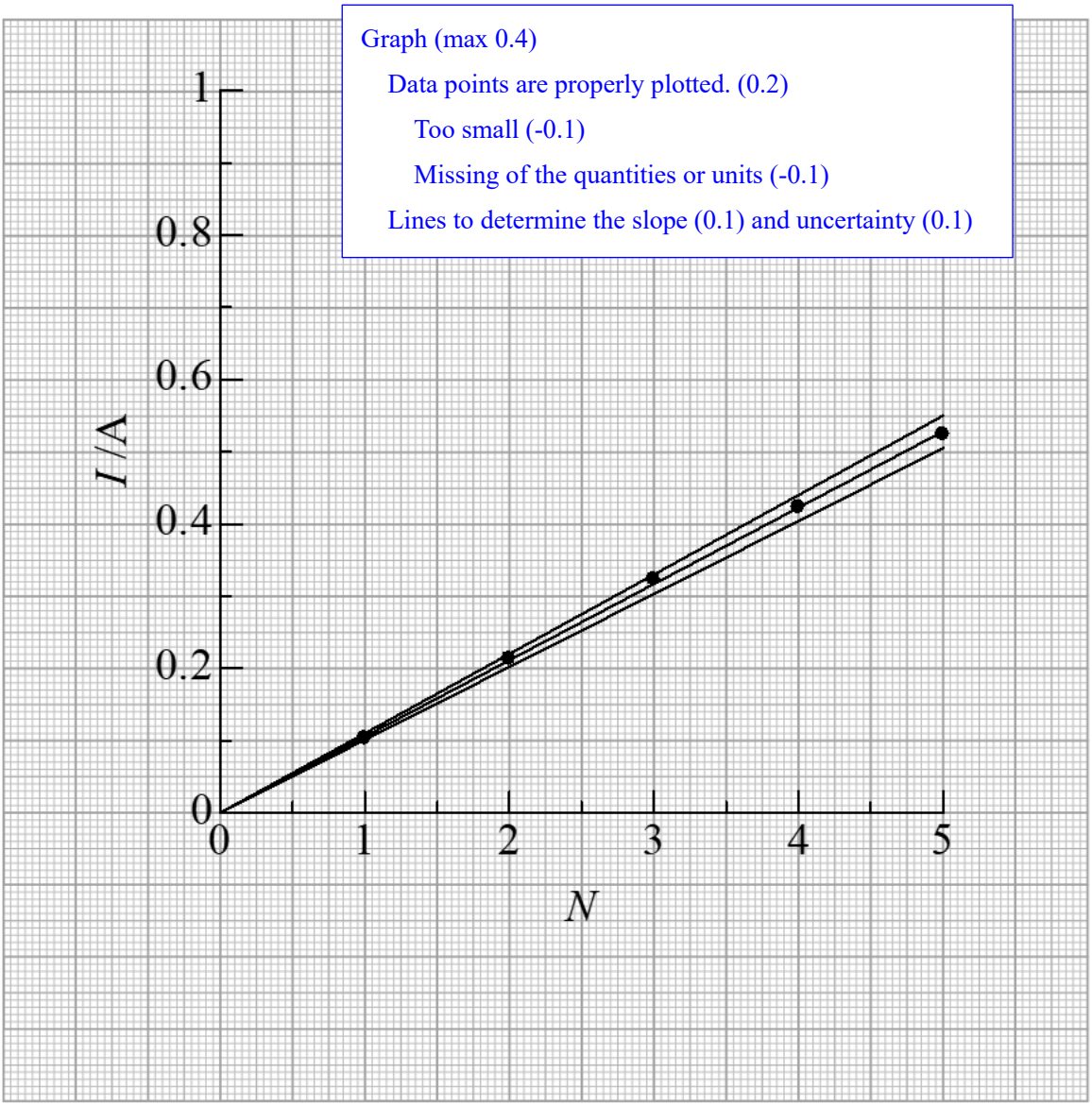
$$a_- = \frac{10.10-12.80}{5} = -0.54$$

$$\Delta a = \frac{-0.48-(-0.54)}{2} = 0.03$$

$$a = -0.51 \pm 0.03 \ \text{mm}$$

Reading of a from the graph (max 0.3)
- Reasonable value (0.1) and uncertainty (0.1)
  - Missing or incorrect units (-0.1)
- Reasonable result of a (correct reading and units required)
  - ~~$-0.08 \leqq a \leqq -0.03$ mm (0.1)~~
  - $-0.8 \leqq a \leqq -0.3$ mm (0.1) Revised

A.4 (0.7pt)



Graph (max 0.4)
- Data points are properly plotted. (0.2)
  - Too small (-0.1)
  - Missing of the quantities or units (-0.1)
- Lines to determine the slope (0.1) and uncertainty (0.1)

$$b = \frac{I}{N} = \frac{0.53}{5} = 0.106$$

$$b_+ = \frac{0.55}{5} = 0.110$$

$$b_- = \frac{0.505}{5} = 0.101$$

$$\Delta b = \frac{0.110 - 0.101}{2} = 0.005$$

$$b = 0.106 \pm 0.005 \ \text{A}$$

Reading of b from the graph (max 0.3)
- Reasonable value (0.1) and uncertainty (0.1)
  - Missing or incorrect units (-0.1)
- Reasonable result of b (correct reading and units required)
  - 0.08–0.13 A (0.1)

Part B: Induced electromotive force (3.0 points)

B.1 (0.2pt)

$V = 2\pi f ABL$

Correct equation (0.2)

B.2 (0.5pt)

$f_B =$ 15.85 Hz

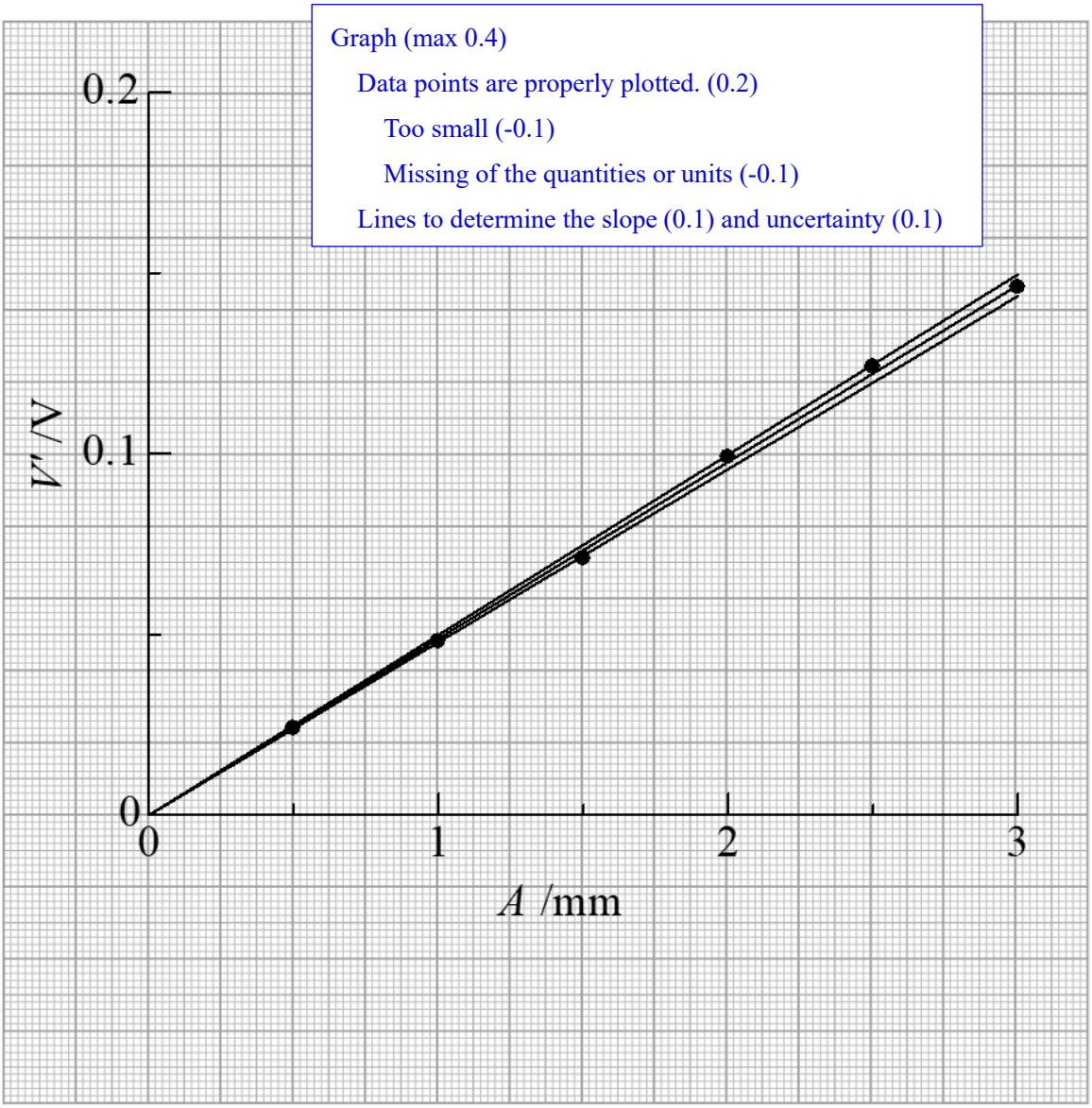
Reasonable result of f and correct units: 12–20 Hz (0.1)

A /mm	V' /V
0.5	0.024
1.0	0.048
1.5	0.071
2.0	0.099
2.5	0.124
3.0	0.146

Measurements of A and V' (max 0.4)
Data points (max 0.3)
5 or more (0.3), 3 or 4 (0.2), 1 or 2 (0.1)
The largest A is 2.5–3.0 mm. (0.1)

B.3 (0.7pt)

Graph (max 0.4)
- Data points are properly plotted. (0.2)
  - Too small (-0.1)
  - Missing of the quantities or units (-0.1)
- Lines to determine the slope (0.1) and uncertainty (0.1)



$$c = \frac{V'}{A} = \frac{0.147}{5} = 0.049 \quad \mathrm{V/mm}$$

$$c_+ = \frac{0.150}{5} = 0.050 \ , \ c_- = \frac{0.144}{5} = 0.048$$

$$\Delta c = \frac{0.050-0.048}{2} = 0.001 \quad \mathrm{V/mm}$$

$$c = 0.049 \pm 0.001 \ \mathrm{V/mm}$$

Reading of c from the graph (max 0.3)
- Reasonable value (0.1) and uncertainty (0.1)
  - Missing or incorrect units (-0.1)
- Reasonable result of c (correct reading and units required)
  - 0.03–0.08 V/mm (0.1)

B.4 (0.4pt)

$BL = \frac{V}{2\pi A f_B}$, $V' = V/\sqrt{2}$

Calculation of BL using the obtained results (max 0.2)
- Correct value and units (0.2), correct calculation formula only (0.1)

Calculation of the uncertainty (max 0.2)
- Correct value (0.2), correct calculation formula only (0.1)

$BL = \frac{\sqrt{2}V'}{2\pi A f_B} = \frac{\sqrt{2}c}{2\pi f_B} = \frac{\sqrt{2}\times 0.049}{2\pi\times 15..85} = 0.000696$ Vs/mm = 0.696 Vs/m

$\Delta(BL) = \frac{\sqrt{2}}{2\pi f_B}\Delta c = \frac{\sqrt{2}\times 0.001}{2\pi\times 15..85} = 0.000014$ Vs/mm = 0.014 Vs/m ($\Delta(BL) = \frac{\boldsymbol{BL}}{c}\Delta c$ available)

$BL = 0.696 \pm 0.014$ Vs/m

B.5 (1.2pt)

$m = \frac{mg}{BL}\cdot\frac{BL}{g} = \frac{I}{N}\cdot\frac{BL}{g} = b\frac{BL}{g} = 0.106 \times \frac{0.696}{9.80} = 0.0075$ kg = 7.5 g

$\Delta m = \sqrt{(\Delta b)^2\cdot\left(\frac{BL}{g}\right)^2 + \left(\frac{b}{g}\right)^2\cdot(\Delta(BL))^2} = 0.00039$ kg = 0.4 g

($\Delta m = \left|\frac{BL}{g}\right|\Delta b + \left|\frac{b}{g}\right|\Delta(BL)$, $\frac{\Delta m}{m} = \frac{\Delta b}{b} + \frac{\Delta(BL)}{BL}$, $\frac{\Delta m}{m} = \sqrt{\left(\frac{\Delta b}{b}\right)^2 + \left(\frac{\Delta(BL)}{BL}\right)^2}$ available)

$m = 7.5 \pm 0.4$ g

Calculation of m using the obtained results (max 0.5)
- Correct value and units (0.2), correct calculation formula only (0.1)
- Reasonable result (correct calculation and units required) (max 0.3)
  - 7.2–8.2 g (0.3), 6.7–8.7 g (0.2), 6.2–9.2 g (0.1)

Calculation of the uncertainty (max 0.2)
- Correct value (0.2), correct calculation formula only (0.1)

$k = -\frac{mg}{a} = -\frac{0.0075\times 9.80}{-0.51} = 0.144$ N/mm =144 N/m

$\Delta k = \sqrt{(\Delta a)^2\cdot\left(\frac{mg}{a^2}\right)^2 + \left(\frac{g}{a}\right)^2\cdot(\Delta m)^2} = 0.011$ N/mm = 11 N/m

($\Delta k = \left|\frac{mg}{a^2}\right|\Delta a + \left|\frac{g}{a}\right|\Delta m$, $\frac{\Delta k}{k} = \frac{\Delta a}{|a|} + \frac{\Delta m}{m}$, $\frac{\Delta k}{k} = \sqrt{\left(\frac{\Delta a}{a}\right)^2 + \left(\frac{\Delta m}{m}\right)^2}$ available)

$k = 144 \pm 11$ N/m

Calculation of k using the obtained results (max 0.3)
- Correct value and units (0.2), correct calculation formula only (0.1)
- Reasonable result (correct calculation and units required):
  - 120–180 N/m (0.1)

Calculation of the uncertainty (max 0.2)
- Correct value (0.2), correct calculation formula only (0.1)

Part C. Mass-dependent resonant frequency (2.3 points)

C.1 (0.2pt)

$$f_N = \frac{1}{2\pi}\sqrt{\frac{k\prime}{M+Nm}}$$

Correct equation (0.2)

C.2 (0.5pt)

Measurements of f (max 0.5)
Missing measurement points (-0.1 each)

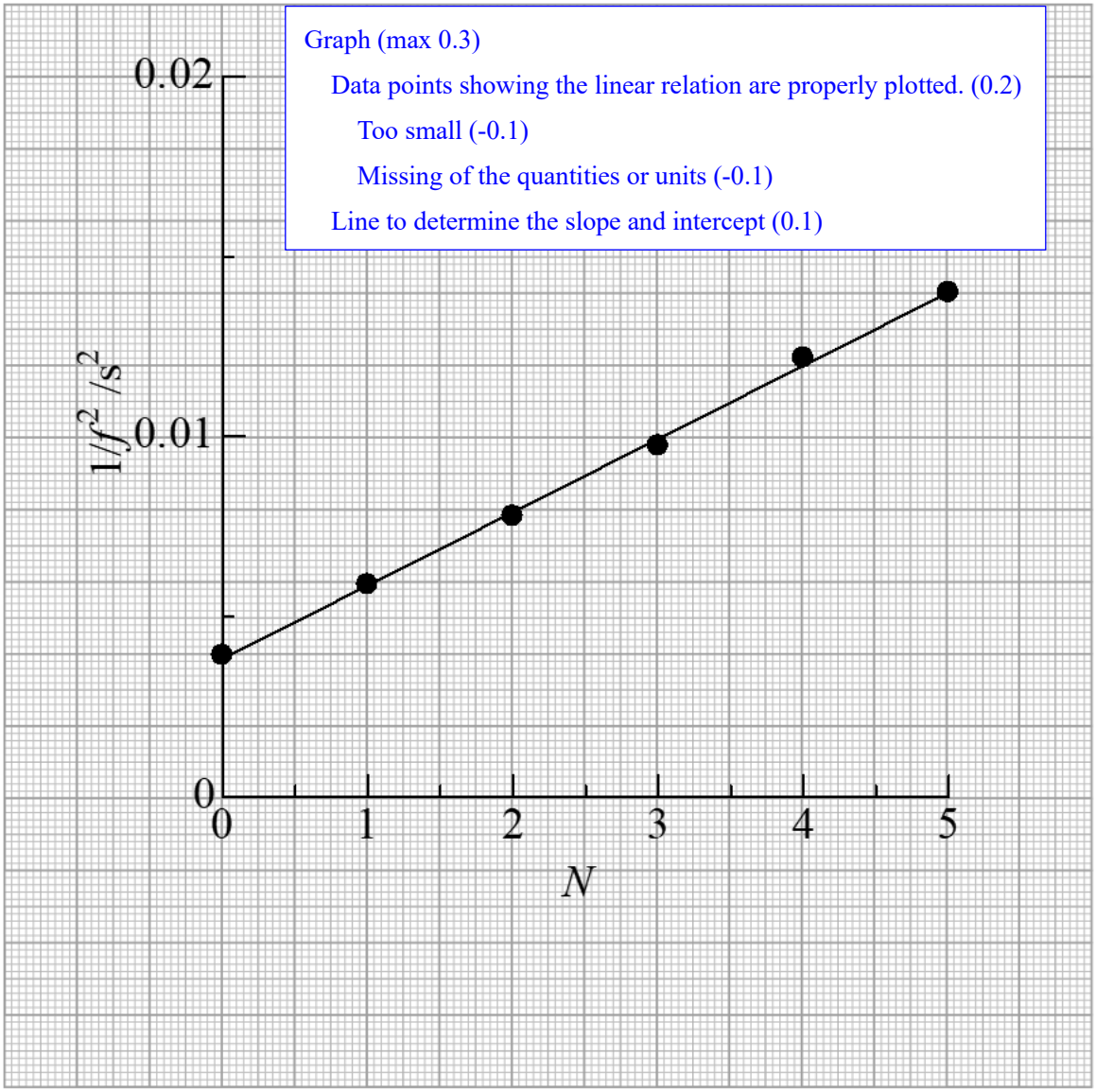
N	f /Hz	$1/f^2$ /s^2		
0	15.96	0.003926		
1	13.03	0.005390		
2	11.33	0.007790		
3	10.13	0.009745		
4	9.06	0.01218		
5	8.45	0.01401		

Calculation for linear relationship (points in C.3)

Calculation for linear relationship ($1/f^2$) in Table C.2 (max 0.3)
Missing or incorrect units (-0.1)
Missing or error of calculation (-0.1 each)

C.3 (1.0pt)

Graph (max 0.3)
Data points showing the linear relation are properly plotted. (0.2)
Too small (-0.1)
Missing of the quantities or units (-0.1)
Line to determine the slope and intercept (0.1)



Using the equation $\frac{1}{f^2} = (2\pi)^2\left(\frac{M}{k'} + \frac{m}{k'}N\right)$, $\frac{M}{k'}$ and $\frac{m}{k'}$ are obtained from the graph.

$$\frac{M}{k'} = \frac{1/f_0^2}{(2\pi)^2} = \frac{0.0039}{(2\pi)^2} = 9.88 \times 10^{-5} \quad \mathrm{s}^2$$

Reading from the graph and calculation (max 0.4)
Reasonable values of $\frac{M}{k'}$ (0.2) and $\frac{m}{k'}$ (0.2)
Missing or incorrect units (-0.1 each)

$$\frac{m}{k'} = \frac{(0.0140-0.0039)/5}{(2\pi)^2} = \frac{0.00202}{(2\pi)^2} = 5.12 \times 10^{-5} \quad \mathrm{s}^2$$

C.4 (0.6pt)

$$\frac{M}{m} = \frac{M/k'}{m/k'} = \frac{9.88}{5.12}$$

$$\frac{M}{m} = 1.93$$

Calculation of $\frac{M}{m}$ using the obtained results (max 0.4)
Correct value and units (0.1)
Reasonable result (correct calculation and units required) (max 0.3)
1.85–2.0 (0.3), 1.75–2.1 (0.2), 1.65–2.2 (0.1)

$$M = \frac{M}{m} \cdot m = 1.93 \times 0.0075 = 0.0145 \text{ kg} = 14.5 \text{ g}$$

$$M = 14.5 \text{ g}$$

Correct value and units of M using the obtained results (0.1)

$$k' = \frac{M}{\frac{M}{k'}} = \frac{0.0145}{9.88\times10^{-5}}$$

$$k' = 147 \text{ N/m}$$

Correct value and units of k' using the obtained results (0.1)

Part D. Resonance characteristics (2.3 points)

D.1 (0.4pt)

$V'_{\mathrm{AC}} = 0.157$ V

Measurement of V'_{AC} and correct units (0.1)

$F_{\mathrm{AC}} = BLI_{\mathrm{AC}} = BL \times 0.106 \times \sqrt{2}V'_{\mathrm{AC}} = 0.696 \times 0.106 \times \sqrt{2} \times 0.157 = 0.0164$ N

Calculation of F_{AC} using the obtained results (max 0.3)
Correct value and units (0.3), correct calculation formula only (0.1)

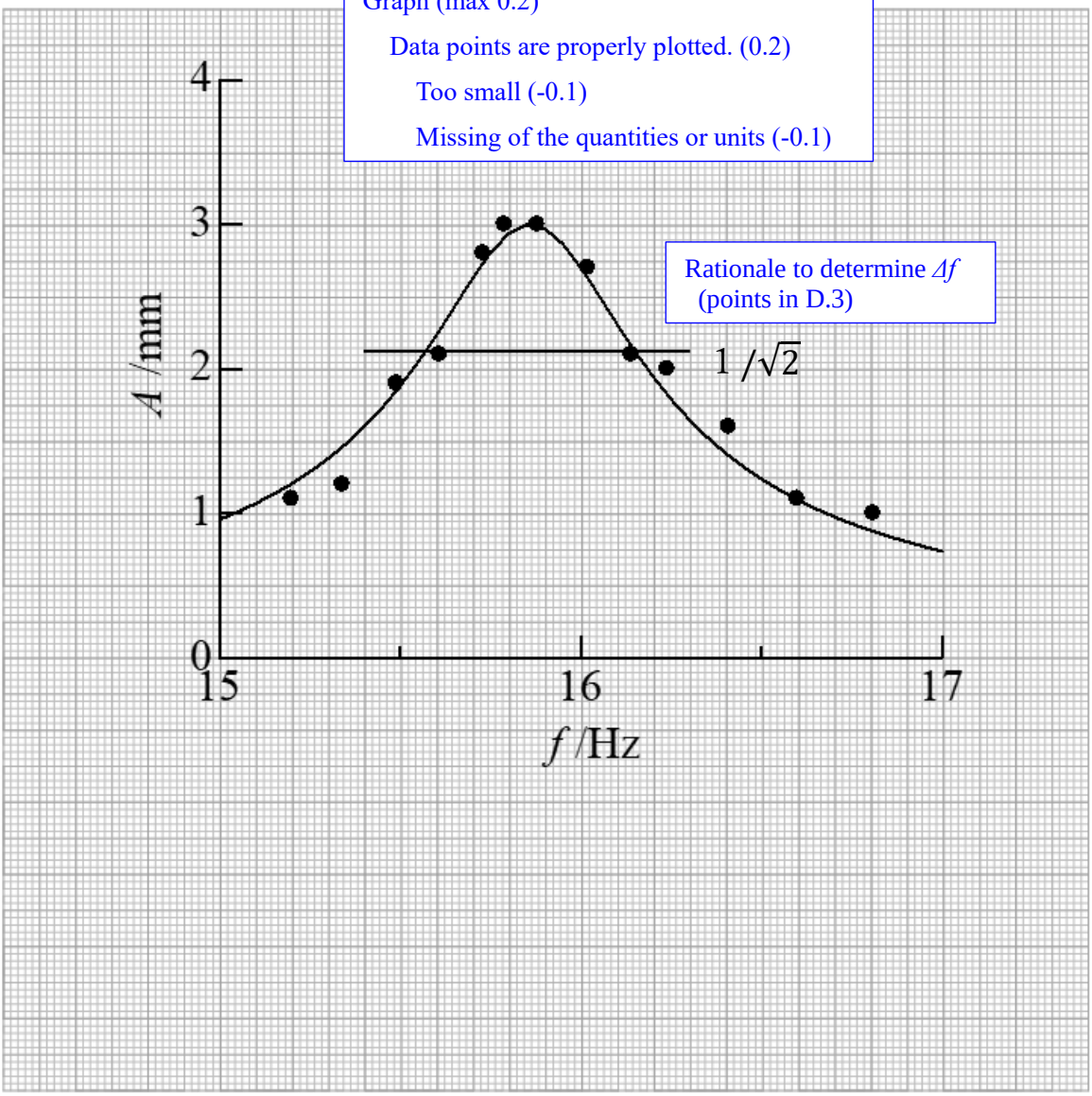
D.2 (0.9pt)

f/Hz	A/mm		
15.88	3.0		
15.79	3.0		
15.73	2.8		
15.61	2.1		
15.49	1.9		
15.34	1.2		
15.20	1.1		
16.02	2.7		
16.14	2.1		
16.24	2.0		
16.41	1.6		
16.60	1.1		
16.81	1.0		

Measurements of f and A (max 0.7)
Data points (max 0.3)
≧10 (0.3), 5–9 (0.2), 3 or 4 (0.1)
Points smaller than half maximum of A (max 0.2: 0.1 each side)
Existence of f interval smaller than 0.2 Hz (0.1)
The largest A is 2.5–3.3 mm. (0.1)

D.2 (cont.)

Graph (max 0.2)
Data points are properly plotted. (0.2)
Too small (-0.1)
Missing of the quantities or units (-0.1)



D.3 (1.0pt)

Reading from the graph D.2

$f_0 = 15.83$ Hz

$A(f_0) = 3.0$ mm

$\Delta f = \frac{16.14-15.56}{2} = 0.29$ Hz

Reading from the graph (max 0.4)
Rationale to determine Δf (0.1)
Reasonable values of f_0, $A(f_0)$, and Δf (0.1 each)

Calculation using Eq.(4)

$$M = \frac{F_{AC}}{8\pi^2 f_0 \Delta f A(f_0)} = \frac{0.0164}{8\pi^2 \times 15.83 \times 0.29 \times 0.003} = 0.0151 \text{ kg} = 15.1 \text{ g}$$

$M = 15.1$ g

Calculation of M using the obtained results (max 0.6)
Correct value and units (0.3), correct calculation formula only (0.1)
Reasonable result (correct calculation and units required) (max 0.3)
13.5–16 g (0.3), 12–17.5 g (0.2), 10.5–19 g (0.1)