IPhO Lithuania 2021

Theoretical Examination

1. Planetary Physics

10 points

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Motivation

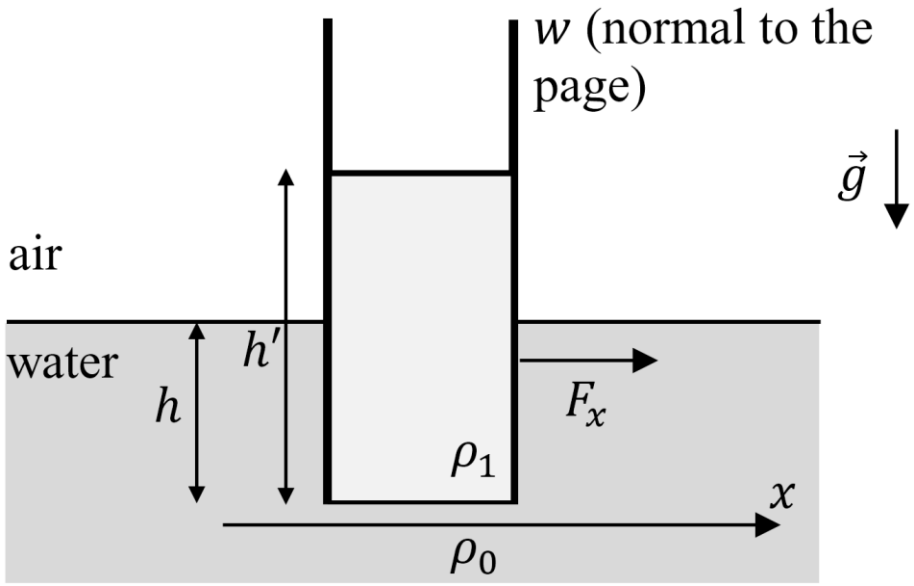
- (Part A) The study of Mid-ocean ridges was the key area of the 20th century geophysical research, giving rise to theory of plate tectonics.
- (A) *Plate cooling model* of a mid-ocean ridge contains many simple physical concepts that are applied to a complex system.
- (Part B) Sophisticated theory of seismic waves can be heavily simplified by using ray theory.
- (B) Measuring seismic wave intensity, including the shadow areas that the waves do not reach, is important when studying the properties of interior of the Earth.

Goals

- To study the plate cooling model of a mid-ocean ridge:
  - Simple initial system that illustrates the key principle of ridge push
  - Thermal expansion of the crust
  - Hydrostatic equilibrium of the ocean + crust + mantle system
  - Ridge push force
  - Thermal timescale of the crust
- To look at a hypothetical scenario of a propagation of waves in a stratified medium:
  - Ray optics with depth-dependent refractive index
  - Studying energy flux emitted by the earthquake
  - Shadow pattern in case of multiple ray reflections

Part A (5 points)
Mid-ocean ridge

- A1 (0.8pt): Consider two vertical plates immersed in water. Oil is poured into the gap between the plates. Find the net horizontal force acting on one of the plates.
  - Balance pressure due to two different columns.
  - Integrate the pressure or find its average to get the force.
- A2 (0.6pt) : Given a linear thermal expansion law for length, find the corresponding law for density of the crust.
  - Asks participants to linearly expand the result.

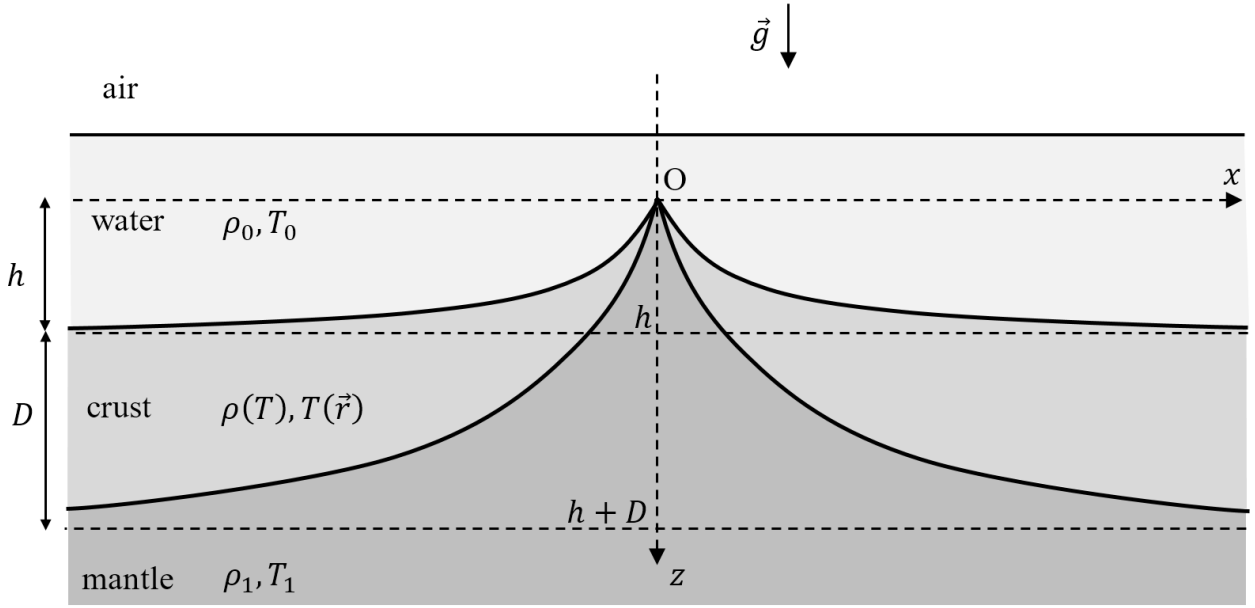


Part A (5 points)
Mid-ocean ridge

- A3 (1.1pt) : Find the far-distance crust thickness in the plate cooling model of a mid-ocean ridge
  - This is a continuum version of the situation in A1: the pressure very deep in the Earth's mantle does not vary in the horizontal direction.
  - Analysis simplified because only verticals at $x = 0$ and $x \to \infty$ have to be considered.

One should equate the deep mantle pressure at the center of the ridge and very far away.

Very far away, the temperature gradient in the crust is constant, resulting in a linear density dependence on height.



Part A (5 points)
Mid-ocean ridge

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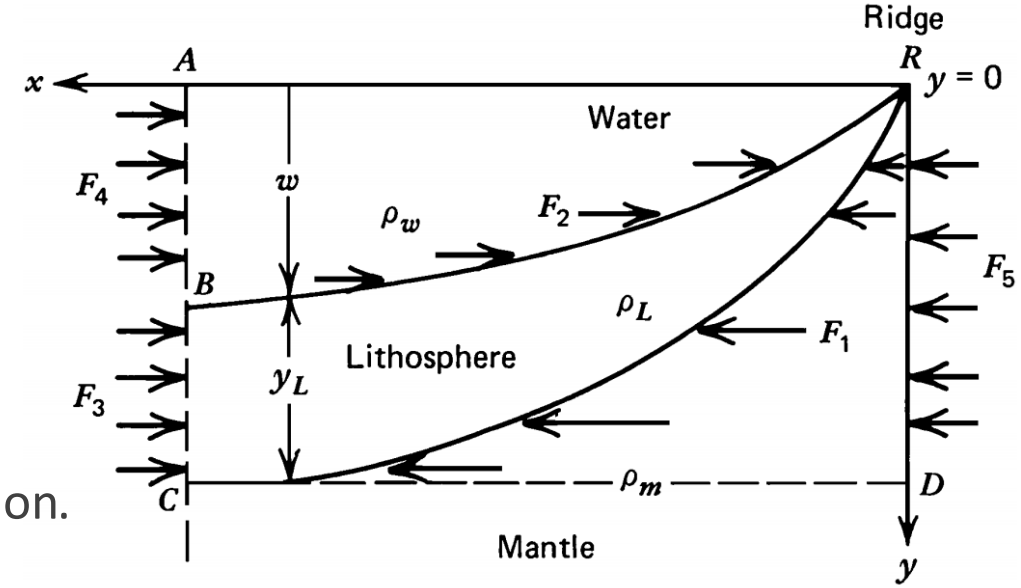
- A4 (1.6pt) : Find the net horizontal force acting on one half of the crust
  - This is the so-called ridge push force.

As in A1, the participants are expected to integrate the pressure at $x = 0$ and $x \to \infty$ to get the two forces acting on the sides of the system. Water and mantle transmit the same forces to the crust.



Turcotte, D.L.; Schubert, G. (2002). "Plate Tectonics". Geodynamics (2 ed.). Cambridge University Press.

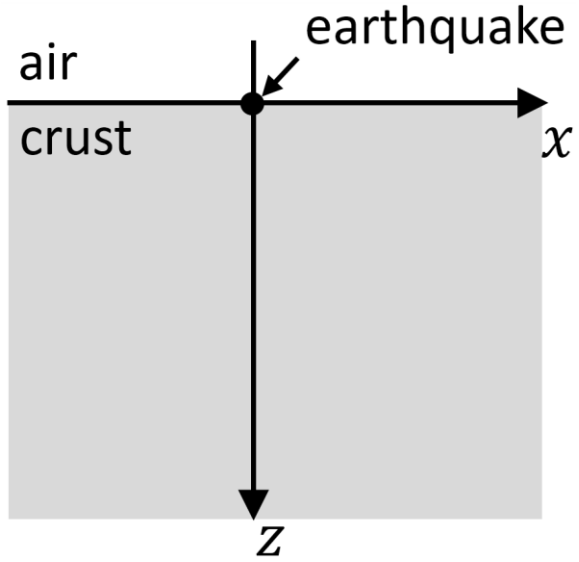
- A5 (0.9pt) : Find the thermal timescale of the crust
  - Exact solution found by solving 1D thermal diffusion equation.
  - Participants are instead asked to use dimensional analysis.

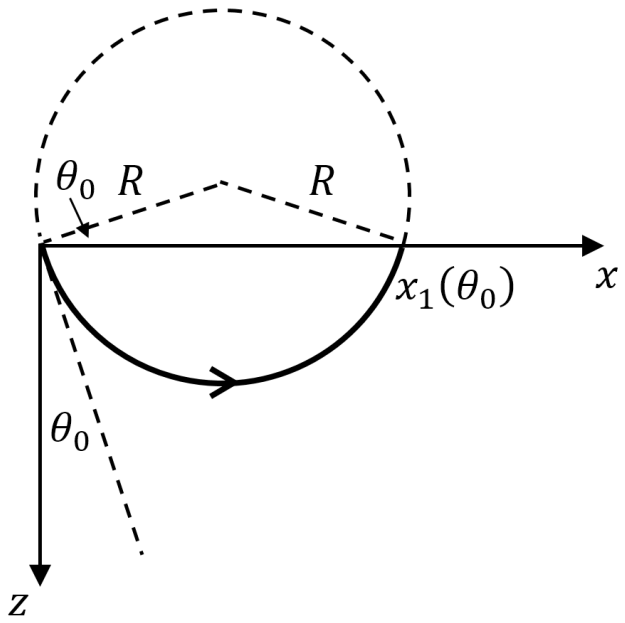
Part B (5 points)
Seismic waves in a stratified medium

- B1 (1.5pt) : At what horizontal distance away from the earthquake does the emitted ray reach the surface?
  - Linear wave speed dependence on depth $v = v_0\left(1 + \frac{z}{z_0}\right)$ results in a ray path being an arc of a circle.

The key idea of the solution is to differentiate the law of refraction,

$$v_0\left(1 + \frac{z}{z_0}\right)\sin\theta_0 = v_0 \sin\theta\,.$$





Part B (5 points)
Seismic waves in a stratified medium

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- B2 (1.5pt): Find how the energy density per unit area $\varepsilon(x)$ absorbed by the surface depends on the distance along the surface x. Plot the result as a function of x.

Since π is the total angle at which the rays are emitted,
$\varepsilon = \frac{E}{\pi}\left|\frac{\mathrm{d}\theta_0}{\mathrm{d}x}\right|$, where $\frac{\mathrm{d}\theta_0}{\mathrm{d}x}$ is found from the B1 result.

- B3 (2.0pt): A device emits a signal at $[\theta_0 - 1/2\ \delta\theta, \theta_0 + 1/2\ \delta\theta]$. If the waves fully reflect from the surface, where is the furthest surface point that the signal does not reach?
  - The surface zones which the signal does reach get broader with increasing x.
  - The increase in the zone width with x can be found by Taylor-expanding the B1 result ($\delta\theta \ll \theta_0$).

Syllabus Coverage

Theoretical skills

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2.1 General

- The ability to make appropriate approximations, while modelling real life problems (A2, A3, B1)

2.2 Mechanics

2.2.2 Statics

- Equilibrium conditions: force balance (vectorially or in terms of projections) (A1, A4)

2.2.5 Hydrodynamics

- Pressure, buoyancy (A1, A2, A3, A4)

Theoretical skills

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2.4 Oscillations and waves

- **2.4.2 Waves**
  - Waves in inhomogeneous media: Snell's law (B1)
  - Energy carried by waves: continuity of the energy flux. (B2)
- **2.4.4 Interaction of electromagnetic waves with matter**
  - Refractive index (B1)
- **2.4.5 Geometrical optics and photometry**
  - Approximation of geometrical optics: rays (B); a full shadow (B3).
  - Luminous flux and its continuity (B2).

Theoretical skills

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2.7 Thermodynamics and statistical physics

- **2.7.2 Heat transfer and phase transitions**
  - Concept of heat conductivity (A5)
  - Continuity of heat flux (A3, A4)

Mathematics

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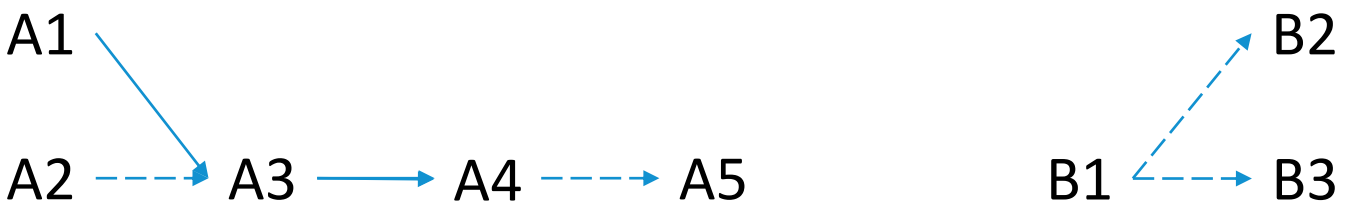
- **4.1 Algebra**
  - Simplification of formulae by factorisation and expansion. Solving linear systems of equations (all A and B)
- **4.2 Functions**
  - Basic properties of trigonometric, inverse-trigonometric functions and polynomials (B)
- **4.7 Calculus**
  - Finding derivatives of elementary functions, their sums, products, quotients, and nested functions (B1, B2)
  - Finding definite and indefinite integrals in simple cases: elementary functions, sums of functions, and using the substitution rule for a linearly dependent argument (A3, A4).
- **4.8 Approximate and numerical methods**
  - Using linear and polynomial approximations based on Taylor series. Linearization of equations and expressions (A2, A4, B2, B3)

Not in the syllabus

- **A.2 – thermal expansion of solids**
  - The relevant equation is given in the problem text, the topic itself is not difficult and is probably covered by school curricula of many countries
- **A.5 – dimensional analysis**
  - A commonly encountered topic in IPhO (e.g., 2016 Problem 3, 2015 Problem 3)

Dependencies



→ many correct ideas of the previous question needed

⇢ correct solution achievable without having solved the previous question

Thank you!