Electrostatic lens (10 points)

General note: if student got the correct answer and the solution is physically and mathematically correct, all the points should be given.

Part A. Electrostatic potential on the axis of the ring (1 point)

A.1 (0.3 pt)	Potential created by a small element of the ring	0.1
	Correct answer	0.2
A.2 (0.4 pt)	Writing the equation in the form that contains a small parameter (z/R)	0.2
	Correct answer	0.2
A.3 (0.2 pt)	Writing the expression for the force	0.1
	Correct sign for q	0.1
A.4 (0.1 pt)	Correct answer	0.1

Part B. Electrostatic potential in the plane of the ring (1.7 points)

B.1 (1.5 pt) via direct integration	Potential created by a small element $\mathrm{d}\varphi$	0.3
	Correct expansion in (r/R)	0.3
	Correct integration	0.6
	Final value for β	0.3
via Gauss's law	The expression of the potential	0.2
	The application of Gauss' law to a cylinder	0.4
	Integral through the side	0.3
	Integral through the base	0.3
	Final value of β	0.3
B.2 (0.2 pt)	Writing the expression for the force	0.1
	Correct sign for q	0.1

Part C. The focal length of the idealized model of an electrostatic lens (2.3 points)

C.1 (1.3 pt)	Writing the equation of motion for r	0.2
	Time of the electron in the “active region”	0.1
	Radial velocity v_r	0.3
	Time to reach the optical axis	0.2
	Focal length f	0.5
C.2 (0.8 pt)	Initial velocity v_r	0.1
	Initial velocity v_z	0.1
	Radial velocity after traversing the “active region”	0.2
	Time to reach the optical axis	0.2
	Expression for c	0.2
C.3 (0.2 pt)	showing that $1/b + 1/c = 1/f$ is valid	0.2

Part C.1 and especially C.2 can be alternatively solved using geometrical considerations (i.e., angles and angles of deflection) rather the “kinematic” arguments presented in the official solution. If the final answer is correct, all points should be given. If the final answer is not correct, but students take an alternative approach, they should be given some points similarly to the table above. For example, calculating the radial velocity after traversing the “active region” is equivalent to find the angle of deflection after traversing the “active” region, so 0.2 points should be give.

Part D. The ring as a capacitor (3 points)

D.1 (2.0 pt)	The idea to separate the ring into two parts	0.4
	Writing the expression for Φ_1	0.2
	Correct value of Φ_1	0.3
	Writing the expression for Φ_2	0.2
	Correct value of Φ_2	0.3
	Correct value of the total potential Φ	0.4
	Correct expression for C	0.2

The marking scheme above pertains to the exact solution. However some students might solve the problem approximately using some quantitative arguments. In this “approximate” solution the students might obtain a capacitance of the form $\frac{4\pi^2\varepsilon_0 R}{\ln\left(\frac{AR}{a}\right)}$, where A is a constant different from 8. If students obtain such an answer, they should be given 1 point.

D.2 (1.0 pt)	Diff. equation for $-d/2v < t < d/2v$	0.1
	Correct answer for $-d/2v < t < d/2v$	0.2
	Correct expression for q_0	0.2
	Diff. equation for $t > d/2v$	0.1
	Correct answer for $t > d/2v$	0.2
	Accurate sketch (0.05 pt per each): charge is negative correct dependence for $t < d/(2v)$ correct dependence for $t > d/(2v)$ additional labels on the axes (q_0, $\pm d/(2v)$)	0.2

Part E. Focal length of a more realistic lens (2.0 points)

E.1 (1.7 pt)	Equation of motion for r	0.2
	Writing v_r as an integral	0.4
	Correct integral from $-d/(2v)$ to $d/(2v)$	0.2
	Correct integral from $d/(2v)$ to ∞	0.2
	Correct expression for v_r	0.2
	Correct focal length	0.5

E.2 (0.3 pt)	Correct answer for q_{eff}	0.3