Theoretical Examination

3. Particles and waves

10 points

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Goal

- Study several aspects of modern physics related to wave–particle duality:
  - De Broglie wave of the particle in a 1D box
  - Energy quantization
  - Photon absorption/emission during the transition between the discrete energy levels
  - Simple model to understand some optical properties of molecules
  - Bose–Einstein condensation
  - Optical lattices as a tool to catch and localize atoms at some specific spatial positions
  - Rydberg atoms

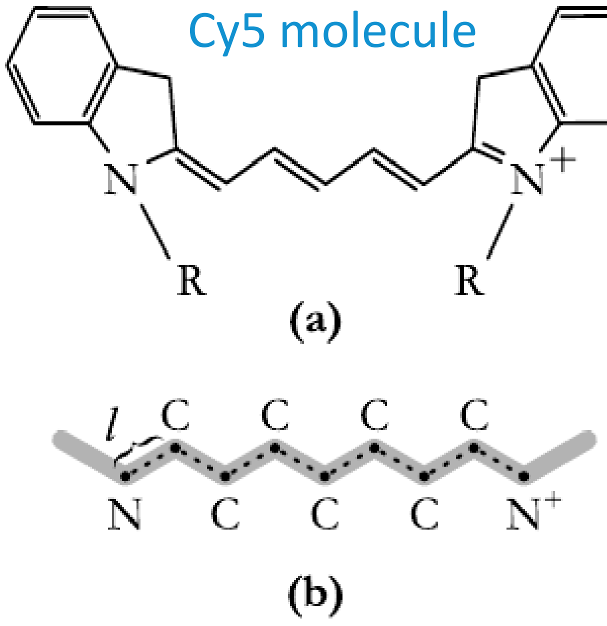
Part A (2 points)
Quantum particle in a box

- Consider a particle in a 1D box ($0 \leq x \leq L$) and describe it as a standing de Broglie wave to determine:
  - A1: ground state energy E_1 (0.3 pt.)
  - A2: probability to detect the ground-state particle within the interval $0 \leq x \leq \frac{1}{4}L$ (1.0 pt.)
  - A3: Energy spectrum E_n (0.4 pt.)
  - A4: Wavelength of the photon emitted during the $E_2 \rightarrow E_1$ transition (0.3 pt.)

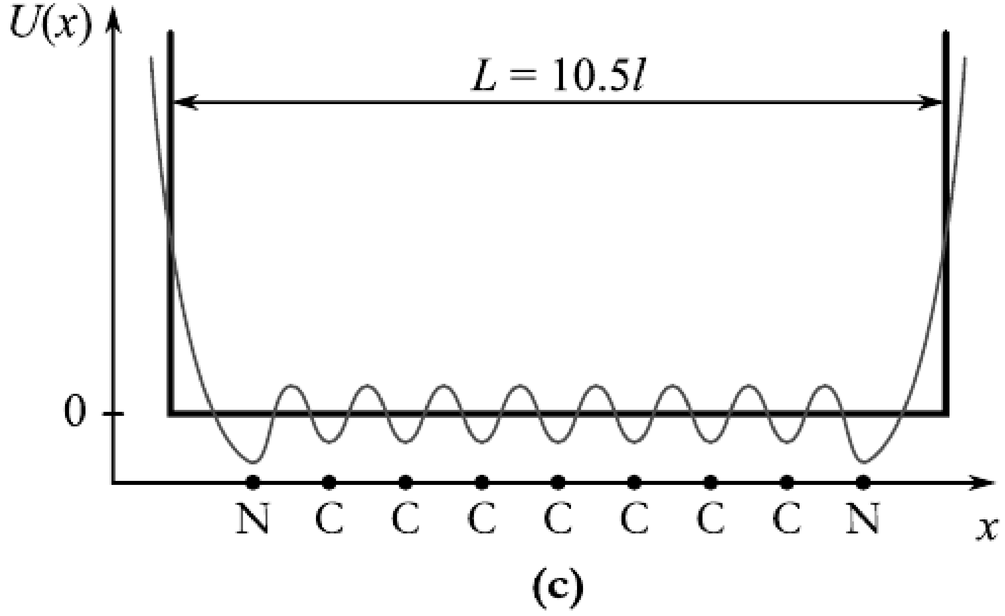
Part B (2 points)
Optical properties of molecules

- Describe cyanine dye molecule as a system of several non-interacting π electrons (“electronic gas”) in a 1D box:

Each atom in the backbone provides a single electron that is “shared” and can move along the whole backbone



Potential energy of the electron along the backbone

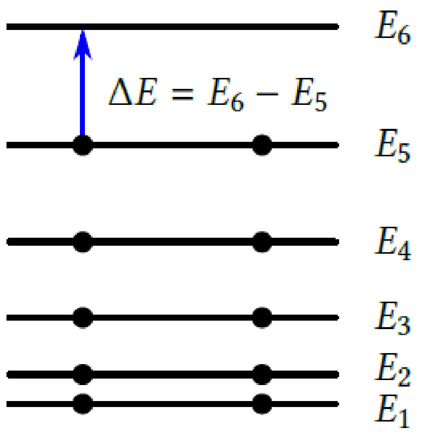


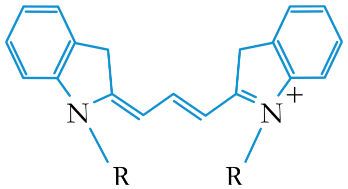
Part B (2 points)
Optical properties of molecules

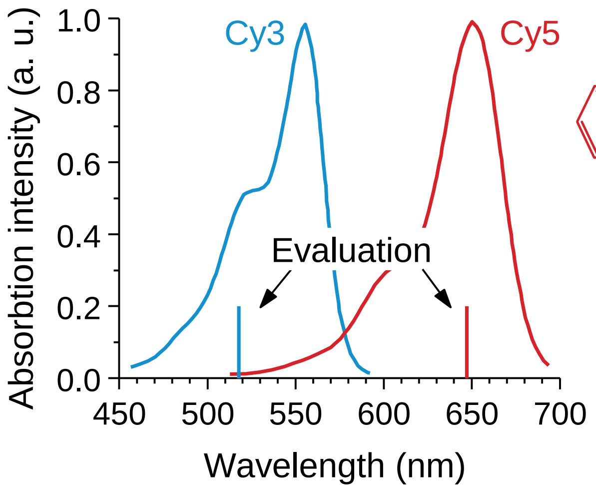
- B1: determine the largest wavelength of the photon that can be absorbed by the Cy5 molecule (0.7 pt.)

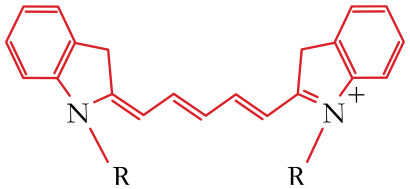
One needs to take into account the Pauli exclusion principle

- B2: evaluate the spectral shift in the absorption spectrum of a shorter Cy3 molecule (0.4 pt.)









Part B (2 points)
Optical properties of molecules

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- B3: determine the expression for the rate of spontaneous transition of a molecule from the first excited to the ground state in terms of ε_0, Planck constant h, emission wavelength λ, and transition electric dipole moment d. (0.7 pt.)

One needs to use dimension analysis to obtain Einstein's coefficient for spontaneous emission $A = \frac{16\pi^3}{3} \cdot \frac{d^2}{\varepsilon_0 h \lambda^3}$ (numerical prefactor is given)

- B4: evaluate the mean fluorescence lifetime of the lowest excited state of Cy5 molecule (0.2 pt.)

$\tau \approx 3.3\ \text{ns}$

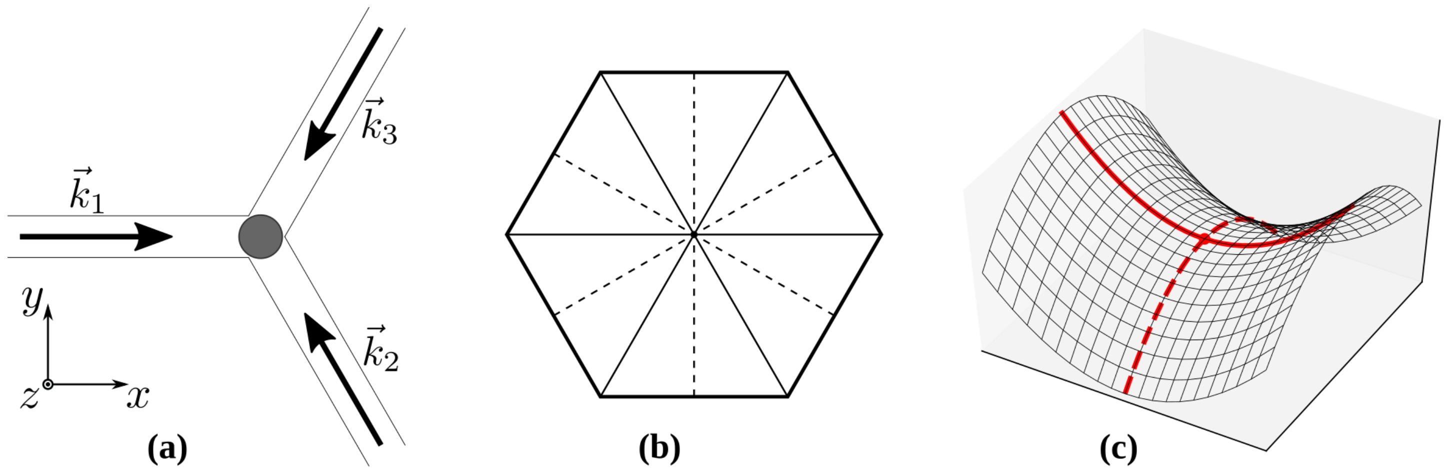
Part C (1.5 points)
Bose-Einstein condensation

- For a bosonic (Rb) quantum gas, explore the observation that Bose-Einstein condensation takes place when the de Broglie wavelength associated with the thermal motion is roughly equal to the inter-particle distance.

  - C1: calculate the temperature-dependent de Broglie wavelength (0.4 pt.)
  - C2: obtain the critical temperature (0.5 pt.)
  - C3: estimate the (surprisingly low) critical density for experimental conditions (0.6 pt.)

Part D (4.5 points)
Three-beam optical lattices

- Explore the notion of optical lattices created by interfering a number of coherent laser beams and providing periodic potential distributions for ultracold atoms. We focus on triangular lattices resulting from interfering three beams.



Part D (4.5 points)
Three-beam optical lattices

- D1: calculate the potential energy distribution $V(\vec{r})$ created by interfering three equal-intensity beams intersecting at 120 degrees (1.0 pt.)

One needs to exploit superposition of fields and perform time-averaging to eliminate rapidly oscillating terms

- D2: argue that the resulting distribution $V(\vec{r})$ is invariant with respect to rotation by 60 degrees (0.5 pt.)

This helpful observation allows to focus on coordinate axes *x* and *y* that run along symmetry lines; no need to plot and study 2D distributions

Part D (4.5 points)
Three-beam optical lattices

- D3: derive the 1D dependences $V_X(x) = V(x, y = 0)$ and $V_Y(x) = V(x = 0, y)$, and perform analysis of extrema (1.0 pt.)

Straightforward search for maxima and minima of single-argument functions

- D4: identify the actual minima of the potential energy, i.e. the lattice sites, figure out the lattice constant, which is the laser wavelength times a numerical factor (0.8 pt.)

Needs to separate true minima of a 2D distribution from saddle points (concept introduced and explained)

- D5: What is the height of the energy barrier separating the adjacent minima? (0.6 pt.)

- D6: Can atoms be as big as the optical wavelength (lattice constant) – YES. (0.6 pt.)

Syllabus Coverage

Theoretical skills

2.1 General

- The ability to make appropriate approximations, while modelling real life problems (B1, B2, D6)
- Recognition of and ability to exploit symmetry in problems (D2, D3, D4)

2.2 Mechanics

2.2.3 Dynamics

- Kinetic energy for translational (A1, A3) and rotational (D6) motions
- Potential energy for simple force fields. Momentum, angular momentum, energy (D6)

2.3 Electromagnetic fields

2.3.2 Integral forms of Maxwell's equations

- Superposition principle for electric field (D1)

Theoretical skills

2.4 Oscillations and waves

2.4.3 Waves

- Propagation of harmonic waves: phase as a linear function of space and time; wave length, wave vector (D1)
- Energy carried by waves: proportionality to the square of the amplitude (A2)

2.4.4 Interference and diffraction

- Superposition of waves (D1)
- Standing waves (A1, A2, A3)

Theoretical skills

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2.6 Quantum Physics

2.6.1 Probability waves

- Particles as waves: relationship between the frequency and energy, and between the wave vector and momentum (A1, A3, C1, C2)
- Energy levels of hydrogen-like atoms (circular orbits only), quantization of angular momentum (D6)

2.6.2 Structure of matter

- Emission and absorption spectra for hydrogen-like atoms (for other atoms —qualitatively) (A4, B1, B2)
- Spectral width and lifetime of excited states (B3, B4)
- Pauli exclusion principle for Fermi particles (B1, B2)

Theoretical skills

2.7 Thermodynamics and statistical physics

2.7.1 Classical thermodynamics

- Kelvin’s temperature scale (C1, C2, C3)
- Translational motion of molecules and pressure (C1)
- Root-mean-square speed of molecules (C1, C3)

Mathematics

4.2 Functions

- Basic properties of trigonometric functions. This includes formulae regarding trigonometric functions of a sum of angles (D1)
- Solving simple equations involving trigonometric functions (D3)

4.4 Vectors

- Basic properties of vectorial sums, dot products (D1)

4.7 Calculus

- Finding derivatives of elementary functions (D3)
- Integration as the inverse procedure to differentiation. Finding definite and indefinite integrals in simple cases: elementary functions, sums of functions, and using the substitution rule for a linearly dependent argument (A2)

Dependencies